Differential Equations

Question1

General solution of the differential equation $\frac{dy}{dx}+y\tan x=\sec x$ is

KCET 2025

Options:

A. $y\sec x=\tan x+c$

B. $y\tan x=\sec x+c$

C. $\operatorname{cosec} x=y\tan x+c$

D. $x\sec x=\tan y+c$

Answer: A

Solution:

$$\frac{dy}{dx}+(\tan x)y=\sec x$$
$$I\cdot F=e^{(-)\int-\frac{\sin x}{\cos x}dx}=e^{-\log_e\cos x}=\sec x$$
$$\therefore\quad y\cdot\sec x=\int\sec^2 x dx$$
$$y\sec x=\tan x+c$$

Question2

If ' a ' and ' b ' are the order and degree respectively of the differentiable equation. $\left(\frac{d^2y}{dx^2}\right)^2+\left(\frac{dy}{dx}\right)^3+x^4=0$, then $a-b=$

KCET 2025

Options:

A. 1

B. 2

C. -1

D. 0

Answer: D

Solution:

Here the highest derivative is $\frac{d^2y}{dx^2}$ so the order $a = 2$. The equation is a polynomial in derivatives and the highest derivative appears as $\left(\frac{d^2y}{dx^2}\right)^2$, so the degree $b = 2$. Hence

$a - b = 2 - 2 = 0.$

Answer: 0.

Question3

The solution of $e^{dy/dx} = x + 1, y(0) = 3$ is

KCET 2024

Options:

A. $y - 2 = x \log x - x$

B. $y - x - 3 = x \log x$

C. $y - x - 3 = (x + 1) \log(x + 1)$

D. $y + x - 3 = (x + 1) \log(x + 1)$

Answer: D

Solution:

$\because e^{\frac{dy}{dx}} = x + 1$

$$\Rightarrow \quad \frac{dy}{dx} = \log(x+1)$$
$$\Rightarrow \quad \int dy = \int \log(x+1)dx$$
$$\Rightarrow \quad y = x\log(x+1) - \int \frac{x}{x+1}dx$$
$$\Rightarrow \quad y = x\log(x+1) - \int 1 \cdot dx + \int \frac{1}{1+x}dx$$
$$\Rightarrow \quad y = x\log(x+1) - x + \log(x+1) + C$$
$$\Rightarrow \quad y = (x+1)\log(x+1) - x + C$$

Now, $y(0) = 3$

$3 = 0 + C \Rightarrow C = 3$

Hence, $y = (x+1)\log(x+1) - x + 3$

$y + x - 3 = (x+1)\log(x+1)$

Question4

The family of curves whose x and y intercepts of a tangent at any point are respectively double the x and y coordinates of that point is

KCET 2024

Options:

A. $xy = C$

B. $x^2 + y^2 = C$

C. $x^2 - y^2 = C$

D. $\frac{y}{x} = C$

Answer: A

Solution:

Let the point on the curve be (h, k).

According to the question, the equation of the tangent will be

$\Rightarrow \quad \frac{x}{2h}+\frac{y}{2k} = 1$

$\Rightarrow \quad \frac{y}{2k} = 1-\frac{x}{2h}$

$\Rightarrow \quad y = -\left(\frac{2k}{2h}\right)x+2k$

So, the slope of tangent will be $\left(-\frac{k}{h}\right)$.

Now, $\frac{dy}{dx}=-\frac{h}{k}=-\frac{y}{x}$

$$\Rightarrow \quad \frac{dy}{y}=-\frac{dx}{x}$$

On integrating both sides, we get

$\int\frac{dy}{dy}=-\int\frac{dx}{x}$

$\Rightarrow \quad \ln y=-\ln x+\ln C$

$\Rightarrow \quad \ln y=\ln\frac{C}{x}$

$\Rightarrow \quad y=\frac{C}{x}$

$\therefore \quad xy=C$

Question5

If a curve passes through the point $(1,1)$ and at any point (x,y) on the curve, the product of the slope of its tangent and x coordinate of the point is equal to the y coordinate of the point, then the curve also passes through the point

KCET 2023

Options:

A. $(3,0)$

B. $(-1,2)$

C. $(\sqrt{3},0)$

D. $(2,2)$

Answer: D

Solution:

Let the equation of the curve by $y = f(x)$ and curve passes through the point $(1, 1)$, So, $f(1) = 1$ Also, we know that at any point (x, y) on the curve, the product of the slope of its tangent and x co-ordinate of the point is equal to the y co-ordinate of the point.

$$\frac{y}{x} = \frac{dy}{dx}$$
$$xy' = y$$

$$\frac{y'}{y} = \frac{1}{x}$$

On integrating both sides w.r.t. x, we get

$$\ln|y| = |\text{In}|x| + e^c$$
$$y = kx, \text{ where } k = e^c$$

Now, we can use the fact that the curve passes through the point $(1, 1)$ to find the value of k.

$\Rightarrow 1 = k(1) \quad k = 1$

Therefore, the equation of the curve is $y = x$

Substituting the value of x and y from the given option, we find that the curve passes through the point $(2, 2)$

Question6

The degree of the differential equation $1 + \left(\frac{dy}{dx}\right)^2 + \left(\frac{d^2y}{dx^2}\right)^2 = \sqrt[3]{\frac{d^2y}{dx^2} + 1}$ is

KCET 2023

Options:

A. 3

B. 1

C. 2

D. 6

Answer: D

Solution:

Given,

$1+\left(\frac{dy}{dx}\right)^2+\left(\frac{d^2y}{dx^2}\right)^2=\sqrt[3]{\frac{d^2y}{dx^2}+1}$

On cubic both sides, we get

$$\left[1+\left(\frac{dy}{dx}\right)^2+\left(\frac{d^2y}{dx^2}\right)^2\right]^3=\left(\frac{d^2y}{dx^2}+1\right)$$
$$\Rightarrow \left[1+\left(\frac{dy}{dx}\right)^2\right]^3+\left(\frac{d^2y}{dx^2}\right)^6+3\left[1+\left(\frac{dy}{dx}\right)^2\right]\cdot\frac{d^2y}{dx^2}$$
$$\left[1+\left(\frac{dy}{dx}\right)^2+\left(\frac{d^2y}{dx^2}\right)^2\right]=\frac{d^2y}{dx^2}+1$$

So, degree of the differential equation is 6 .

Question7

If $\frac{dy}{dx}+\frac{y}{x}=x^2$, then $2y(2)-y(1)=$

KCET 2022

Options:

A. $\frac{11}{4}$

B. $\frac{15}{4}$

C. $\frac{9}{4}$

D. $\frac{13}{4}$

Answer: B

Solution:

$\frac{dy}{dx}+\frac{y}{x}=x^2$

On comparing to $\frac{dy}{dx}+P(x)y=Q(x)$

$$P(x)=\frac{1}{x}, Q(x)=x^2$$
$$\mathrm{IF}=e^{\int P dx}=e^{\int \frac{1}{x} dx}=e^{\ln x}=x$$
$$y \cdot(\mathrm{IF})=\int Q(x)(\mathrm{IF}) dx+C$$
$$y \cdot x=\int x^2 x dx+C=\frac{x^4}{4}+C$$
$$y=\frac{x^3}{4}+\frac{C}{x}$$
$$y(2)=\frac{2^3}{4}+\frac{C}{2}=2+\frac{C}{2}$$
$$y(1)=\frac{1}{4}+\frac{C}{1}=\frac{1}{4}+C=x$$
$$2y(2)-y(1)=2\left(2+\frac{C}{2}\right)-\left(\frac{1}{4}+C\right)$$
$$=4+C-\frac{1}{4}-C=\frac{15}{4}$$

Question8

The solution of the differential equation $\frac{dy}{dx}=(x+y)^2$ is

KCET 2022

Options:

A. $\tan^{-1}(x+y)=x+C$

B. $\tan^{-1}(x+y)=0$

C. $\cot^{-1}(x+y)=C$

D. $\cot^{-1}(x+y)=x+C$

Answer: A

Solution:

$\frac{dy}{dx}=(x+y)^2$ ..... (i)

Let $x+y=t \Rightarrow 1+\frac{dy}{dx}=\frac{dt}{dx}$

Eq. (i) can be written as

$$\frac{dt}{dx} - 1 = t^2$$

$$\Rightarrow \quad \frac{dt}{dx} = t^2 + 1 \Rightarrow \frac{dt}{t^2+1} = dx$$

$$\Rightarrow \int \frac{dt}{t^2+1} = \int dx \Rightarrow \tan^{-1} t = x + C$$

$$\Rightarrow \quad \tan^{-1}(x+y) = x + C$$

Question9

If $y(x)$ is the solution of differential equation $x \log x \frac{dy}{dx} + y = 2x \log x, y(e)$ is equal to

KCET 2022

Options:

A. e

B. 0

C. 2

D. 2e

Answer: C

Solution:

$$x \log x \frac{dy}{dx} + y = 2x \log x \Rightarrow \frac{dy}{dx} + \frac{y}{x \log x} = 2$$

$$\text{IF } = e^{\int \frac{1}{x \log x} dx} = e^{\log(\log x)} = \log x$$

$$y(\log x) = \int (2)(\log x) \cdot dx + C$$

$$\Rightarrow \quad y \log x = 2 \int \log x dx + C$$

$$\Rightarrow \quad y \log x = 2[x \log x - x] + C \quad \text{.... (i)}$$

Putting $x = 1$, we get

$y \cdot 0 = 2[1 \times 0 - 1] + C \Rightarrow C = 2$

Putting $C = 2$ in Eq. (i), we get

$y \log x = 2[x \log x - x] + 2$

At $\quad x = e \Rightarrow y \cdot 1 = 2(e \times 1 - e) + 2$

$y = 0 + 2 = 2$

$y(e) = 2$

Question10

The sum of the degree and order of the differential equation $\left(l + y_1^2\right)^{2/3} = y_2$ is

KCET 2022

Options:

A. 4

B. 6

C. 5

D. 7

Answer: C

Solution:

$$\left(1 + y_1^2\right)^{2/3} = y_2$$
$$\Rightarrow \quad \left[\left(1 + y_1^2\right)^{2/3}\right]^3 = (y_2)^3 \Rightarrow \left(1 + y_1^2\right)^2 = y_2^3$$

Here, highest derivative is y_2.

Therefore, order is 2 and power of y_2 is 3.

So, the degree is 3.

Hence, required sum $= 2 + 3 = 5$

Question11

Solution of differential equating $xdy - ydx = 0$ represents

KCET 2021

Options:

A. A rectangular hyperbola.

B. Parabola whose vertex is at origin.

C. Straight line passing through origin.

D. A circle whose centre is origin.

Answer: C

Solution:

Given, $xdy = ydx$

$\Rightarrow \quad \frac{1}{y}dy = \frac{1}{x}dx$

Integrating both sides,

$$\int \frac{1}{y}dy = \int \frac{1}{x}dx.$$

$\Rightarrow \quad \log y = \log x + \log c$

$\Rightarrow y = xc$ which is the equation of line passing through origin.

Question12

The number of solutions of $\frac{dy}{dx} = \frac{y+1}{x-1}$, when $y(1) = 2$ is

KCET 2021

Options:

A. three

B. one

C. infinite

D. two

Answer: B

Solution:

$$\frac{dy}{dx} = \frac{y+1}{x-1}$$
$$\Rightarrow \quad \frac{1}{x-1}dx = \frac{1}{y+1}dy$$

Integrating both sides,

$$\int \frac{1}{x-1}dx = \int \frac{1}{y+1}dy$$
$$\Rightarrow \quad \log|x-1| = \log|y+1| + \log C$$
$$\Rightarrow \quad (x-1) = c(y+1)$$

As,
$y(1) = 2$
$0 = c(3)$
$c = 0$

Hence, $x - 1 = 0$ or $x = 1$

The given differential equation has only one solution.

Question13

The order of the differential equation obtained by eliminating arbitrary constants in the family of curves $c_1 y = (c_2 + c_3)e^{x+c_4}$ is

KCET 2020

Options:

A. 1

B. 2

C. 3

D. 4

Answer: A

Solution:

We have, $c_1 y=(c_2+c_3)e^{x+c_4}$

$$\Rightarrow \quad y=\left(\frac{c_2+c_3}{c_1}\right)e^{c_4}\cdot e^x$$

$$\Rightarrow \quad y=ce^x \text{ where } c=\frac{c_2+c_3}{c_1}e^{c_4}$$

Taking log on both sides, we get

$$\log y=\log\left(ce^x\right)$$

$$\Rightarrow \quad \log y=\log c+x\log e$$

$$\Rightarrow \quad \log y=\log c+x$$

On differentiating w.r.t. x, we get

$\frac{1}{y}\frac{dy}{dx}=1\Rightarrow\frac{dy}{dx}=y$

$\therefore$ order $=1$

Question14

The general solution of the differential equation $x^2dy-2xydx=x^4\cos xdx$ is

KCET 2020

Options:

A. $y=x^2\sin x+cx^2$

B. $y=x^2\sin x+c$

C. $y=\sin x+cx^2$

D. $y=\cos x+cx^2$

Answer: A

Solution:

We have,

$$x^2 dy - 2xy dx = x^4 \cos x dx$$
$$\Rightarrow \quad \frac{dy}{dx} - \frac{2}{x} y = x^2 \cos x$$

Comparing with $\frac{dy}{dx} + Py = Q$

Where $P = -\frac{2}{x}$ and $Q = x^2 \cos x$.

$$\therefore \quad \text{IF} = e^{\int \frac{-2}{x} dx} = e^{-2 \log x} = e^{\log\left(\frac{1}{x^2}\right)} = \frac{1}{x^2}$$

$\therefore$ The solution of the given differential

$$y \cdot \text{IF} = \int (Q \times \text{IF}) dx + c$$
$$\Rightarrow \quad y \times \frac{1}{x^2} = \int \left(x^2 \cos x\right) \left(\frac{1}{x^2}\right) dx + c$$
$$\Rightarrow \quad \frac{y}{x^2} = \int \cos x + c$$
$$\Rightarrow \quad \frac{y}{x^2} = \sin x + c$$
$$\Rightarrow \quad y = x^2 \sin x + cx^2$$

Question15

The curve passing through the point $(1, 2)$ given that the slope of the tangent at any point (x, y) is $\frac{3x}{y}$ represents

KCET 2020

Options:

A. Circle

B. Parabola

C. Ellipse

D. Hyperbola

Answer: D

Solution:

We have,

$$\frac{dy}{dx}=\frac{3x}{y}$$

$$\Rightarrow \quad \int y dy=\int 3x dx$$

$$\Rightarrow \quad \frac{y^2}{2}=3\cdot\frac{x^2}{2}+C \quad \text{.... (i)}$$

Since, Eq (i) passing through point $(1,2)$

$$\therefore \quad \frac{4}{2}=\frac{3}{2}(1)^2+c$$

$$c=2-\frac{3}{2}=\frac{1}{2}$$

$\therefore$ Equation of curve, $\frac{y^2}{2}=\frac{3x^2}{2}+\frac{1}{2}$

$$\Rightarrow y^2=3x^2+1 \Rightarrow y^2-3x^2=1$$

$$\Rightarrow \frac{y^2}{(1)^2}-\frac{x^2}{(1/\sqrt{3})^2}=1$$

Which represents a Hyperbola.

Question16

The integrating factor of the differential equation $\left(2x+3y^2\right)dy=ydx(y>0)$ is

KCET 2019

Options:

A. $\frac{1}{x}$

B. $\frac{1}{e^y}$

C. $\frac{1}{y^2}$

D. $-\frac{1}{y^2}$

Answer: C

Solution:

We have differential equation,

$(2x+3y^2)dy = ydx, y>0$

$$\Rightarrow \quad y\frac{dx}{dy} = 2x+3y^2 \Rightarrow \frac{dx}{dy} - \frac{2x}{y} = 3y$$

Which is linear differential equation in the form of $\frac{dx}{dy} + Px = Q$.

$\therefore \quad I.F = e^{\int Pdy} = e^{\int \frac{-2}{y}dy} = e^{-2\log y} = e^{\log\left(\frac{1}{y^2}\right)} = \frac{1}{y^2}$

Question17

The equation of the curve passing through the point $(1,1)$ such that the slope of the tangent at any point (x,y) is equal to the product of its co-ordinates is

KCET 2019

Options:

A. $2\log y = x^2 - 1$

B. $2\log x = y^2 - 1$

C. $2\log x = y^2 + 1$

D. $2\log y = x^2 + 1$

Answer: A

Solution:

According to the question, slope of tangent at any point (x,y) = product of its co-ordinates

$$\Rightarrow \quad \frac{dy}{dx} = xy \Rightarrow \frac{dy}{y} = x \cdot dx \Rightarrow \int \frac{dy}{y} = \int x dx$$

$$\Rightarrow \quad \log y = \frac{x^2}{2} + C \quad \text{.... (i)}$$

Eq. (i) passing through $(1,1)$

$\therefore \quad \log 1 = \frac{1}{2} + C \Rightarrow C = -\frac{1}{2}$

$\therefore$ required equation of curves is

$\log y = \frac{x^2}{2} - \frac{1}{2} \Rightarrow 2\log y = x^2 - 1$

Question18

The order of the differential equation $y = C_1 e^{C_2 + x} + C_3 e^{C_4 + x}$ is

KCET 2019

Options:

A. 3

B. 1

C. 4

D. 2

Answer: B

Solution:

We have, $y = C_1 e^{C_2 + x} + C_3 e^{C_4 + x}$

$\Rightarrow y = C_1 e^{C_2} e^x + C_3 e^{C_4} e^x \Rightarrow y = A \cdot e^x + B \cdot e^x$

(Here, $A = C_1 e^{C_2}, B = C_3 e^{C_4}$)

$\Rightarrow y = (A + B)e^x = Ce^x$ (Here, $A + B = C$)

Here, number of arbitrary constant is one so, order of the differential equation is one.

Question19

The degree and the order of the differential equation $\frac{d^2y}{dx^2} = \sqrt[3]{1 + \left(\frac{dy}{dx}\right)^2}$ respectively are

KCET 2018

Options:

A. 2 and 3

B. 3 and 2

C. 2 and 2

D. 3 and 3

Answer: B

Solution:

To find the degree and the order of the differential equation, consider the given equation:

$$\frac{d^2y}{dx^2} = \sqrt[3]{1 + \left(\frac{dy}{dx}\right)^2}$$

To simplify the equation, cube both sides:

$$\left(\frac{d^2y}{dx^2}\right)^3 = 1 + \left(\frac{dy}{dx}\right)^2$$

Now, identify the order and degree of the equation. The **order** is determined by the highest derivative present, which in this case is $\frac{d^2y}{dx^2}$. Therefore, the order is 2.

The **degree** is determined by the power of the highest derivative after the equation has been expressed in polynomial form (if possible). Since the highest power of $\frac{d^2y}{dx^2}$ in this equation is 3, the degree is 3.

Thus, the differential equation has a degree of 3 and an order of 2.

Question20

The solution of the differential equation $x\frac{dy}{dx} - y = 3$ represents a family of

KCET 2018

Options:

A. straight lines

B. circles

C. parabolas

D. ellipses

Answer: A

Solution:

To solve the differential equation $x\frac{dy}{dx} - y = 3$, we start by rewriting it:

$x\frac{dy}{dx} - y = 3$

Rearrange the terms to isolate the derivatives:

$x\,dy = (y+3)\,dx$

Separate variables to integrate both sides:

$\frac{dy}{y+3} = \frac{dx}{x}$

Now, integrate both sides:

$\int \frac{dy}{y+3} = \int \frac{dx}{x}$

After integration, we have:

$\log(y+3) = \log x + \log c$

This can be rewritten as:

$\log(y+3) = \log(cx)$

Exponentiating both sides, we find:

$y + 3 = cx$

This equation, $y = cx - 3$, represents a family of straight lines.

Question21

The integrating factor of $\frac{dy}{dx} + y = \frac{1+y}{x}$ is

KCET 2018

Options:

A. xe^{x}

B. $xe^{1/x}$

C. $\frac{e^x}{x}$

D. $\frac{x}{e^x}$

Answer: C

Solution:

To solve for the integrating factor, we start with the differential equation:

$\frac{dy}{dx} + y = \frac{1+y}{x}$

By manipulating the equation, it can be rewritten as:

Subtract $\frac{y}{x}$ from both sides:

$\frac{dy}{dx} + y - \frac{y}{x} = \frac{1}{x}$

Rearrange to:

$\frac{dy}{dx} + (1 - \frac{1}{x})y = \frac{1}{x}$

In this form, we identify:

$P = 1 - \frac{1}{x}$

$Q = \frac{1}{x}$

Next, we find the integrating factor (I.F.), given by:

$\text{I.F.} = e^{\int P\,dx}$

Calculate the integral of P:

$\int P\,dx = \int (1 - \frac{1}{x})\,dx = \int 1\,dx - \int \frac{1}{x}\,dx = x - \log x$

Substituting back into the expression for the integrating factor:

$\text{I.F.} = e^{(x - \log x)} = \frac{e^x}{e^{\log x}} = \frac{e^x}{x}$

Thus, the integrating factor is $\frac{e^x}{x}$.

Question22

General solution of differential equations

$\frac{dy}{dx} + y = 1 (y \neq 1)$ **is**

KCET 2017

Options:

A. $\log\left|\frac{1}{1-y}\right| = x + C$

B. $\log|1-y| = x + C$

C. $\log|1+y| = x + C$

D. $\log\left|\frac{1}{1-y}\right| = -x + C$

Answer: A

Solution:

We have,

$$\begin{aligned} & \frac{dy}{dx} + y = 1 \\ \Rightarrow \quad & \frac{dy}{dx} = 1 - y \\ \Rightarrow \quad & \frac{1}{1-y} dy = dx \end{aligned}$$

On integrating both the sides, we have

$$\int \frac{1}{1-y} dy = \int dx$$

$$\Rightarrow \quad -\log|1-y| = x + C$$

$$\Rightarrow \quad \log\left|\frac{1}{1-y}\right| = x + C$$

Question23

The degree of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right]^2 = \frac{d^2y}{dx^2}$ is

KCET 2017

Options:

A. 3

B. 2

C. 1

D. 4

Answer: C

Solution:

We have,

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^2=\frac{d^2y}{dx^2}$$

Since, power of $\frac{d^2y}{dx^2}$ is 1 .

$\therefore$ Degree of the given differential equation is 1 .

Question24

The integrating factor of the differential equation

$x\cdot\frac{dy}{dx}+2y=x^2$ **is** $(x\neq 0)$

KCET 2017

Options:

A. x

B. $\log|x|$

C. x^2

D. $e^{\log x}$

Answer: C

Solution:

To solve the differential equation, we start with:

$x\frac{dy}{dx} + 2y = x^2$

Rewriting it in standard linear form, we have:

$\frac{dy}{dx} + \frac{2}{x}y = x$

This is a linear differential equation, and to solve it, we need to find the integrating factor. The integrating factor, $\mu(x)$, is given by:

$\mu(x) = e^{\int \frac{2}{x}\,dx}$

Calculating the integral, we get:

$\mu(x) = e^{2\log x}$

Using the properties of logarithms and exponents:

$e^{2\log x} = e^{\log x^2} = x^2$

Thus, the integrating factor is x^2.
